A Practical Introduction to Electricity
—
Direct Current

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Contents

The purpose of the following experiments is to introduce the techniques used for measuring direct current (DC) quantities as well as the general procedures generally used in practical lab work. As most students are not familiar with the physics of electricity a brief theoretical introduction is given at the beginning of each unit. The practical work should be properly documented, the documentation being a important aspect of the issue.

1. Resistor and Resistance

1.1. Learning Objectives

The assignment of this practical work is to measure and to draw the current-voltage characteristic of an common resistor up to the maximal power allowed. This might not be very exciting at first glance...
The students shall get to know the following physical quantities, features, and concepts:

- electrical *resistance* as the *property* of a resistor
- *nominal*, *measured*, and "*real*" value of a physical quantity
- *linearity*, Ohm's law stating the proportionality between current and voltage
- the degree of *influence of temperature* on the resistance
- acquaintance with basic measuring techniques

1.2. Theoretical Introduction

By definition the ratio R between the voltage U and the current I of a resistor (Widerstand als Objekt) is called **resistance** (Widerstand als Eigenschaft). The reciprocal ratio G, i.e. current vs. voltage, is called **conductance** (Leitwert als Eigenschaft):

$$R = \frac{U}{I} \qquad \text{with unit } [R] = \frac{V}{A} = \Omega \text{ or ohm} \tag{1.1}$$

$$G = \frac{I}{U} \qquad \text{with unit } [G] = \frac{A}{V} = \Omega^{-1} \text{ or mho} \tag{1.2}$$

The resistance is dependent on the *geometry* and the *substance* of the resistor as well as on its *temperature*. Kept at constant temperature the resistance of metals and most semiconductors will be independent of the current or the voltage applied. This is called **Ohm's law**.

Ohm's law
At constant temperature the current in metals is proportional to the applied voltage:

$$I \propto U \tag{1.3}$$

In an adequately chosen domain the dependency of the resistance R from the temperature θ (given in degrees Centigrade) can be described by the following linear equation:

$$R = R_{20}\big(1 + \alpha_{20}(\theta - 20°\mathrm{C})\big) \tag{1.4}$$

$$\frac{\Delta R}{R_{20}} = \frac{R - R_{20}}{R_{20}} = \alpha_{20}(\theta - 20°\mathrm{C}) = \alpha_{20}\Delta\theta$$

The parameter α_{20} is called **temperature coefficient** and refers here to a temperature of 20°C, R_{20} being the resistance value at that temperature. The value of the coefficient is dependent on the reference temperature! A positive value means that the resistance will increase with temperature (PTC: **p**ositive **t**emperature **c**oefficient), a negative one that it will decrease (NTC: **n**egative **t**emperature **c**oefficient). Metals show usually PTC and semiconductors NTC behavior. These features are widely used for temperature measurement, e.g. PT100: http://en.wikipedia.org/wiki/Resistance_thermometer.

1.3. Practical Part

The task consists of measuring the U-I-Characteristic of a resistor with 100 Ω and 5 Watt nominal values. The following figure shows the schematics:

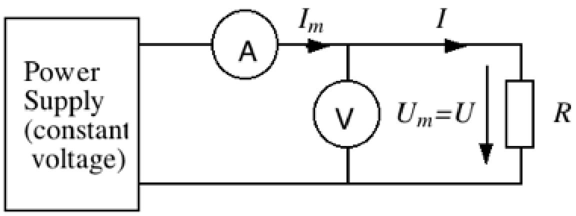


Figure 1.1.: Measurement Circuit
The connectors of the voltmeter should be attached directly to the resistor to avoid catching the voltage dropping in the line. Due to the voltmeter resistance the measured current I_m is not exactly equivalent to the current I in the resistor. This error however remains negligible due to the comparatively (compared to R) high resistance of the voltmeter.

The voltage of the power supply should be increased step by step from low to higher values, selecting about 10 regularly separated points. The maximal allowed value at the resistor can be calculated with the following formula:

$$U_{max} = \sqrt{P_{max} \cdot R_{nominal}}$$

While P_{max} is the maximal dissipated power in the resistor at ambient temperature.

Changing the voltage changes the current and also the temperature. The latter will not react immediately to the changes. Try to find out how long the temperature needs to settle to the next higher value and make sure to wait a while before going to the next point in order to see the influence of the temperature on the resistance. Note that the time to heat is the same as the time to cool down.

1.4. Evaluation

In order to evaluate the measurements it is necessary to list the possible **objectives** of the experiment.[1] *These are not equivalent to the assignment and are usually not explicitly given*! You can imagine your own or make a choice out of the following suggestions and related questions:

- How much does the "true" value of the resistor differ from its nominal value? Are the deviations within the allowed margins given by the manufacturer ($\pm 5\%$ tolerance)?
- Does the resistor behave according to Ohm's law?
- Does the resistance depend on the current? How? How much?
- How does the increasing part of the resistance depend on the dissipated power?
- Are there fluctuations due to uncertainty in the measurement? How strong are these?
- What is the typical settling time of the temperature?

All the answers should be given in a generalized form, for example uncertainties in % of the nominal value of the resistance.

For recording the measured values and working out graphically the different relations between current, voltage, resistance, and power you may use the following Matlab program:[2]

[1] The objectives of en experiment shall not be confounded with the learning objectives!
[2] This program can be inserted in the Matlab Editor via copy-paste.

Listing 1.1: Matlab program for recording measurement data and plotting different resistor characteristics

```
% ETEK, Spring 2013
% Characteristic of a 100 Ohm / 5 Watt Resistor
%
% Copyright 2012, Martin Schlup
%
clear all, clc, format compact

% Nominal Values
Rn=100;  % resistance in Ohm
Pmax=5;  % max. power in Watt
Imax=sqrt(Pmax/Rn)  % max. current in Ampere
Umax=Rn*Imax  % max. voltage in Volt
I=linspace(0,Imax); % current values for plot of nominal characteristic
U=Rn*I; % corresponding voltages

% measurement (fictive)
Um=[2.00 4.00 6.00 8.00 10.0 12.0  14.0 16.0 18.0 20.0 22.0];  % in V
Im=[19.9 39.6 60.0 78.9 99.0 119   138  157  176  193  211];  % in mA

% graphical representation
figure(1)
plot([0 Im],[0 Um],'ob',1e3*I,U,'b')  % measured values drawn as circles
axis([0 1.1e3*Imax 0 1.1*Umax]) % limits of plot
title('Characteristic of a 100 \Omega / 5 W Resistor')
xlabel('\rightarrow  {\it I}  in mA')
ylabel('\rightarrow  {\it U}  in V')
grid
legend('Measured','Nominal',4)  % legend in 4th quadrant

% computed resistance
Rm=Um./Im;  % in kOhm
Rm=1e3*Rm;  % in Ohm

% computed dissipated power
Pm=Um.*Im;  % in mW
Pm=1e-3*Pm;  % in W

figure(2)
plot(Im,Rm,'ob',1e3*I,Rn*ones(size(I)),'b')
title('Resistance of a 100 \Omega / 5 W Resistor')
xlabel('\rightarrow  {\it I}  in mA')
ylabel('\rightarrow  {\it R}  in \Omega')
grid
legend('Measured','Nominal',4)
```

2. Source Characteristics

2.1. Learning Objectives

The assignment of this unit is to measure and to record the characteristic of a real voltage source and to compare it to linearized models.
The students shall get to know the following terms, features, and concepts:

- difference between *active* and *passive* two-terminal devices (Zweipole)
- *linear idealization* of voltage and current sources (voltage drop proportional to current)
- *open circuit voltage* (Leerlaufspannung), *short-circuit current* (Kurzschlussstromstärke)
- *maximum obtainable power* (Leistungsanpassung) and *load adjustment* for linear sources (Leistungsanpassung)

2.2. Theoretical Introduction

Real voltage sources such as batteries and accumulators do usually not behave as ideal sources. They generally show an increasing drop in voltage with increasing load current. The relation between the voltage U and the current I can mostly be approximated by a linear function:

$$U = U_0 - \frac{U_0}{I_0} I = U_0 - R_i I \tag{2.1}$$

$$I = I_0 - \frac{I_0}{U_0} U = I_0 - G_i U \tag{2.2}$$

The equations (2.1) and (2.2) describe so called *linear sources*. The parameters U_0 and I_0 being the **open circuit voltage** (Leerlaufspannung) and the **short-circuit current** (Kurzschlussstromstärke) of the source. The ratio $R_i = \frac{U_0}{I_0}$ corresponds to the **internal resistance** of the source, while the reciprocal value is its **conductance** $G_i = \frac{1}{R_i} = \frac{I_0}{U_0}$. A linear source can be realized with an *ideal voltage source* with a resistor or an *ideal current source* with a resistor see fig 2.1 below:

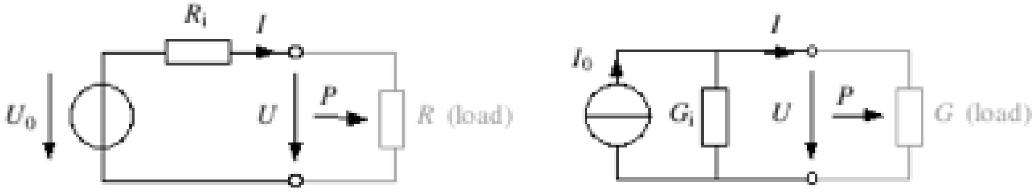


Figure 2.1.: Equivalent linear sources realized as voltage (left) or current source (right)

Both realizations are equivalent in so far that their behavior cannot be distinguished at the connectors, that is when considering the voltage U, the current I, and the energy flow (power) $P = UI$ at the connectors (see figure 2.2). It is generally useful to utilize the conductances instead of the resistances when the elements are connected in parallel as in the case of the current source.

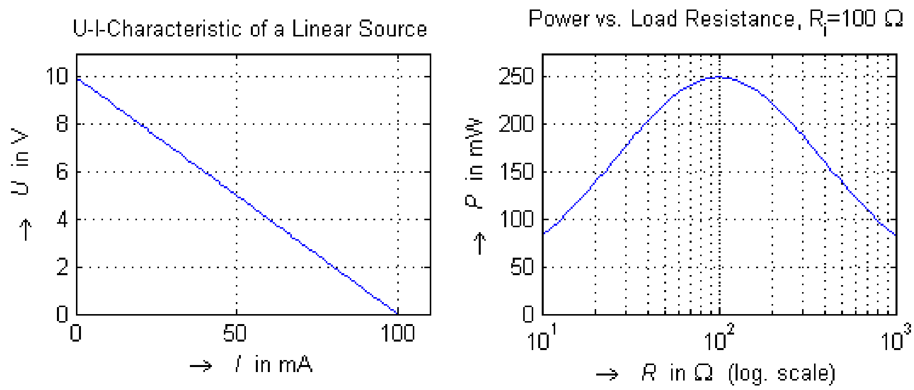


Figure 2.2.: Characteristic and Power Delivery of a Linear Source
(open circuit voltage: $U_0 = 10\,\mathrm{V}$, short-circuit current: $I_0 = 100\,\mathrm{mA}$,
maximum delivered power at load resistance $R = R_i = 100\,\Omega$: $P_{max} = 250\,\mathrm{mW}$)
Note that the load resistance is drawn on a logarithmic scale, yielding a symmetric curve.

Linear sources have an optimal **operating point** (Arbeitspunkt) at which the power delivered to the load is maximum. This point on the U-I-characteristic is at half the open circuit voltage and half the short-circuit current, i. e. at a load resistance equivalent to the inner resistance of the source. *This last statement applies only to linear sources*!

2.3. Practical Part

2.3.1. Linear Sources

The U-I-Characteristics of a linear voltage and a linear current source shall be measured and compared. The figures 2.3 and 2.4 show the schematics. The inner resistor $R_i = 1/G_i$ with nominal resistance $100\,\Omega$ should be the same for both sources. Make sure that the open-circuit voltages U_0 and the short-circuit currents I_0 of the two sources fit as well as possible considering the real (measured) resitance of the inner resistor. Choose something between 8 and 10 V for U_0. Make sure that the maximal power does not exceed the specifications of both the inner and the load resistors.

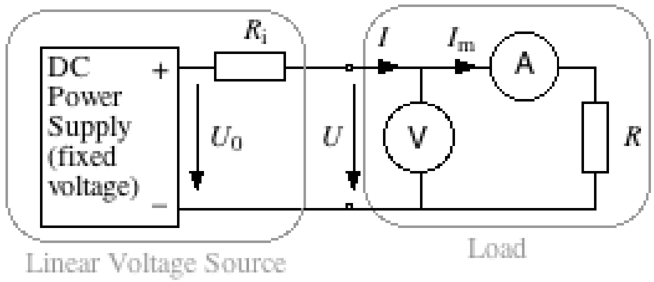


Figure 2.3.: Measurement Circuit for Linear Voltage Source
The source is simulated with a DC power supply and a resistor connected in series. The connectors of the voltmeter should be attached directly to the source to avoid catching the voltage dropping in the line. The load is realized by the variable load resistance and the two measuring instruments: $R_{load} = \frac{U}{I_m}$. The open-circuit voltage U_0 can be measured at the disconnected DC power spply.

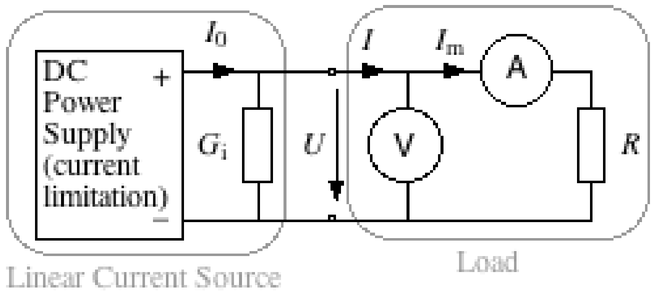


Figure 2.4.: Measurement Circuit for Linear Current Source
The source is simulated with a DC power supply and a resistor connected in parallel. The connectors of the voltmeter should be attached directly to the source to avoid catching the voltage dropping in the line. The load is realized by the variable load resistance and the ampere meter: $R_{load} = \frac{U}{I_m}$. The short-circuit current I_0 must be measured at the disconnected DC power supply (i. e. without the resistor G_i) with the ampere meter only.

2.3.2. Battery

The task consists of measuring the U-I-Characteristic of a 4.5 V cell. The figure 2.5 shows the schematics.

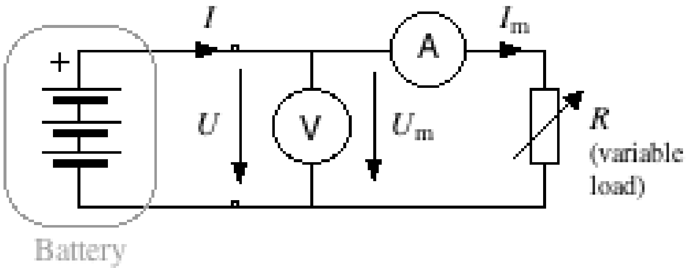


Figure 2.5.: Measurement Circuit
The connectors of the voltmeter should be attached as close as possible to the battery contacts to avoid catching the voltage dropping in the connecting line. Due to the voltmeter resistance the measured current I_m is not exactly equivalent to the current I in the resistor. This bias error however remains negligible due to the comparatively high resistance of the voltmeter. The load is realized by the variable load resistance and the ampere meter: $R_{load} = \frac{U}{I_m}$. The short-circuit current can not be measured, as the ampere meter resistance is not zero.

In order not to overload the load resistor the maximum delivered power from the battery should be estimated first. Expecting a short-circuit current of about $I_0 \approx 5\,\mathrm{A}$ and a open circuit voltage of $U_0 \approx 4.5\,\mathrm{V}$ for a new battery of type 3LR12 or 3R12, the maximal delivered power would be for load resistances of about $1\,\Omega$:

$$P_{max} = \frac{U_0}{2}\frac{I_0}{2} \approx 6\,\mathrm{W}.$$

That means that the load resistors should sustain at least 5 W and that the battery should be connected to the load for short times only.

2.4. Evaluation

List of possible objectives of the experiment or related questions:

Linear sources

- Draw the expected U-I-characteristics and the measured points in a common U-I-graph for both sources. How well do these characteristics fit? Are there practical reasons if they don't?

  It is very important here to estimate the uncertainties in measurement for this question to explain possible differences between the model (equation) and the measured points. In a first step the systematical errors should be identified and possibly eliminated.
- Can one distinguish the two source types looking at the characteristics only?
- Why are the slopes of the two characteristics identical?
- Work out the formula for $P = f(R)$ and draw the expected curve and the measured points in a P-R-graph (see figure 2.2, right side). Try to work out for which load resistance one would get maximal power out of both source types?

  Again explain the possible differences between the model (equation) and the measured points with uncertainties in measurement.
- How could the condition for maximum delivered power to a resistive load be stated for nonlinear source characteristics, that is with no defined inner resistance? Does the load resistance be linear to fit the condition?

Battery

- How well can the characteristic of the battery be approximated by an idealized linear source?
- Which parameters such as open circuit voltage and internal resistance would fit the characteristic of the battery best?
- At which theoretical load resistance would one get maximal power out of the battery?
- Why is the linear voltage source a better physical description of he battery behavior than the linear current source?

All the questions should be answered and commented based on the measurements. Furthermore *an estimation of measurement accuracy should be made for all the results.*

3. Superposition Principle

3.1. Learning Objectives

The assignment of this unit is to investigate the simulated behavior of a power generator coupled to a back-up battery using different calculation methods.
The students shall get to know and apply the following laws and concepts:

- *Kirchhoff's voltage law* and *Kirchhoff's current law*
- concept of *reference direction* and *reference arrow*
- *principle of superposition*
- computing *equivalent resistances* for serial and parallel connections

3.2. Theoretical Introduction

A power generator coupled to a back-up battery can be modeled as two linear voltage sources, that is with a couple of ideal voltage sources and inner resistors. The schematic of such a simple model together with a resistive load is shown in figure 3.1.

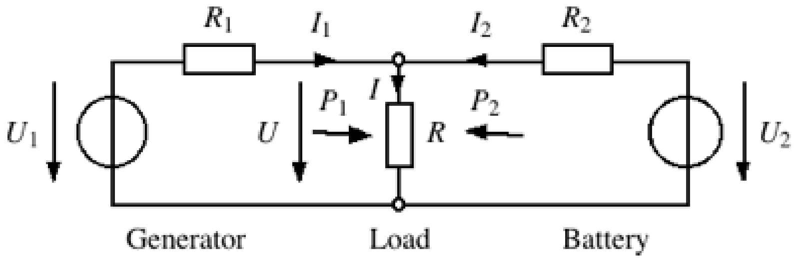


Figure 3.1.: Schematic of a DC-generator (U_1, R_1) connected to a load R and a back-up battery (U_2, R_2)
The reference directions (arrows P_1 and P_2) for the power point both outward of the generator and the battery in the direction of the load resistor. This is an arbitrary choice.
It is assumed that during full charge operation the generator provides part of its power to an uncharged battery in order to reload it. Note that in this case the current I_2 and the Power P_2 must be negative and that $U_1 > U > U_2$ holds. During charging the voltage U_2 of the battery will slowly rise to the voltage U of the load so that the current I_2 will eventually be zero. This latter situation is not rendered in the model as the voltage of the second source is kept constant.

In order to compute all the voltages and currents for a given circuit one needs a set of linearly independent equations. These are provided by the constitutive equations (Materialgesetz) giving the relationship between voltage and current for all the resistors and the laws of balance (Bilanzgesetz) for voltages and current, i. e. the Kirchhoff voltage and current laws (Maschen- und Knotensatz).

For the circuit of figure 3.1 the constitutive and the Kirchhoff laws yield the following set of equations (considering the set reference directions of the voltages and currents):

Constitutive Laws :

$$U_{R1} = R_1 I_1 \tag{3.1}$$

$$U_{R2} = R_2 I_2 \tag{3.2}$$

$$U = RI \tag{3.3}$$

Voltage Balance Laws :

$$U_1 - U_{R1} - U = 0 \tag{3.4}$$

$$U_2 - U_{R2} - U = 0 \tag{3.5}$$

Current Balance Law :

$$I_1 + I_2 - I = 0 \tag{3.6}$$

With these equations, the given values of U_1, U_2 and the known resistances it should be possible to find the solution for all other variables. However due to the linearity of the circuit, there exists an easier way to solve this problem with two sources: using the principle of superposition (Superpositionsprinzip).

> In linear systems the overall action of several independent sources can be additively built from the actions of all the sources taken separately.

Therefore the current or the voltage in the load resistor can be computed in two steps assuming in each case that only one of the two ideal voltage sources is active, then adding the results. Being inactive for an ideal voltage source means that the voltage is zero so that the source acts like a short-circuit (cf. figures 3.2 and 3.3). An inactive ideal current source would look like a open connection with no current flowing.

If the source of the generator is active and the one of the battery inactive the schematic is reduced to the following:

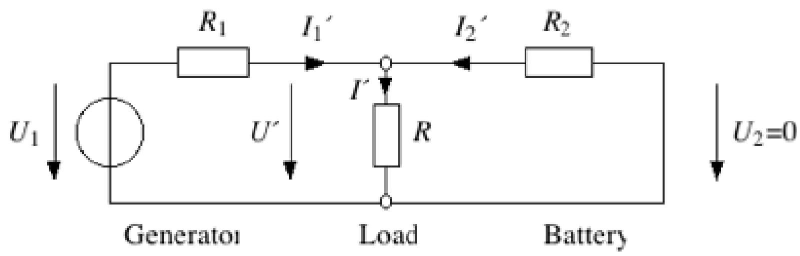


Figure 3.2.: Schematic of the circuit with only the source U_1 working ($U_2 = 0\,\mathrm{V}$)

The current I' for the circuit of figure 3.2 can be computed as follows (using the equivalent resistance of the whole circuit and the current divider rule):

$$I' = \frac{U_1}{R_1 + \frac{RR_2}{R+R_2}} \cdot \frac{R_2}{R + R_2} = \frac{U_1 R_2}{R_1 R_2 + R(R_1 + R_2)}$$

If the source of the battery is active and the one of the generator inactive the schematic is reduced to the following:

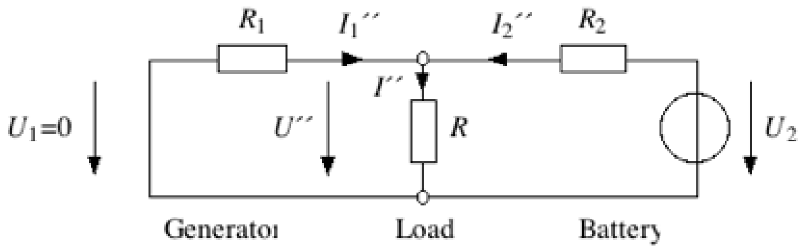


Figure 3.3.: Schematic of the circuit with only the source U_2 working ($U_1 = 0\,\mathrm{V}$)

The formula for the current I'' of the circuit 3.3 can be taken from the formula for I' by exchanging the indices:

$$I'' = \frac{U_2 R_1}{R_1 R_2 + R(R_1 + R_2)}$$

Applying the principle of superposition yields for the case of both sources working:

$$\begin{aligned} I &= I' + I'' = \frac{U_1 R_2 + U_2 R_1}{R_1 R_2 + R(R_1 + R_2)} \\ U &= RI \end{aligned}$$

The same procedure can be applied to all the other currents and voltages in the circuit.

3.3. Practical Part

In order to simulate a battery being loaded ($P_2 < 0$) with an active voltage source, the circuit of figure 3.1 must be completed with the resistor R_0 (see figure 3.4 for details).

Choose the following values for the elements of the circuit:

$$\begin{aligned} U_1 &= 12.0\,\mathrm{V} \\ U_2 &= 10.5\,\mathrm{V} \\ R_1 &= 100\,\Omega \\ R_2 &= 100\,\Omega \\ R &= 1.0\,\mathrm{k}\Omega \\ R_0 &= 3.3\,\mathrm{k}\Omega \end{aligned}$$

Verify through computing that the dissipated power in all the resistors does not exceed the allowed limits, that the current I_3 in the source U_2 is positive, and that I_2 is negative. Make

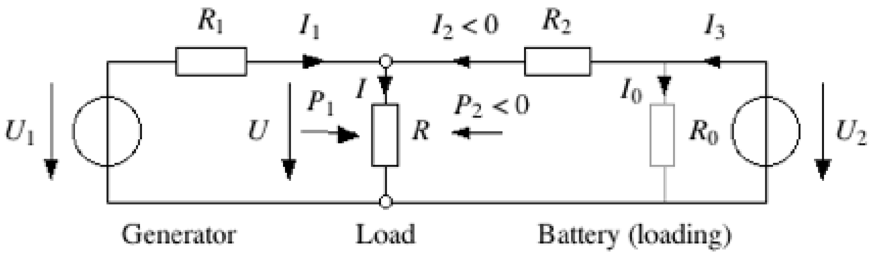


Figure 3.4.: Schematic for the Simulation of a Loading Battery
Loading the battery means that the current I_2 must flow to the right, i. e. be negative, while I_3 *must be positive because the lab power supply realizing the ideal source of the battery cannot be operated when absorbing energy.* This can be achieved by an additional resistor $R_0 < \frac{U_2}{|I_2|}$. This resistor does not influence the behavior of the circuit (since the current $I_0 = \frac{U_2}{R_0}$ depends only on the voltage U_2), but ensures the proper operating of the voltage source simulating the power consumption of the battery.

also sure that the balance of power holds, especially that he generator P_1 feeds all the resistors and the simulated battery. Measure all the voltages of the circuit after having switched on the two sources and compare the results with the expectations before continuing. The currents shall be determined through measuring the voltage over the resistors.

Make sure that the sum of the partial results applying the superposition principle gives the voltage over the load. For making the individual sources inactive you will have to remove them, and to short-circuit the corresponding connectors of the circuit.

3.4. Evaluation

List of possible objectives of the experiment or related questions:

- How well does the principle of superposition hold? Does it hold for all voltages and currents in all the components?

  Note: The circuit should be calculated using Matlab for solving Kirchhoff's laws for the unknown voltages and currents (see example 3.5 and listing 3.1).

  It is important here to estimate the uncertainties in measurement to explain possible differences between the calculated and the measured values.

- At which value of the resistor R_0 will the current I_3 be zero?

- Investigate how the load voltage changes with the battery charge condition.

- Which power would the charged battery deliver to the load resistor when the generator is disconnected?

All the questions should be answered and commented based on the computed results reinforced by measurements. Furthermore *an estimation of measurement accuracy should be made for all the results.*

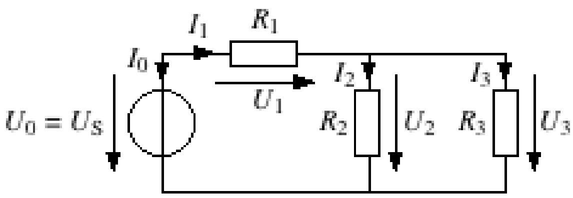


Figure 3.5.: Simple Resistor Circuit with Given Values for the Resistances R_i and the Source Voltage U_s
Note that for computation every element in this example has its own voltage and current defined even if this seems to be unnecessary at first glance.

Listing 3.1: Matlab Program Example for Calculating Resistor Circuits (brute force approach)

```
% Matlab Program Example for Calculating Resistor Circuits
%
% Copyright 2014, Martin Schlup
%
clear all, clc, format compact

% Given Values
R1=50;    % resistance in Ohm
R2=200;   % resistance in Ohm
R3=300;   % resistance in Ohm
Us=10;    % source voltage in Volts

% unknown voltages and currents Ui and Ii for i=0,1,2,3
% equation for the source                 U0           = Us
% equations for the resistances Ri        Ui - Ri*Ii   = 0  for i=1,2,3
% Kirchhoff voltage laws                 -U0 + U1 + U2 = 0
%                                        -U2 + U3      = 0
% Kirchhoff current laws                 -I0 - I1      = 0
%                                         I1 - I2 - I3 = 0

%  U0  U1  U2  U3  I0  I1  I2  I3
A=[ 1   0   0   0   0   0   0   0
    0   1   0   0   0  -R1  0   0
    0   0   1   0   0   0  -R2  0
    0   0   0   1   0   0   0  -R3
   -1   1   1   0   0   0   0   0
    0   0  -1   1   0   0   0   0
    0   0   0   0  -1  -1   0   0
    0   0   0   0   0   1  -1  -1 ];
b=[Us   0   0   0   0   0   0   0 ]';

x=A\b;
U=x(1:4)
I=x(5:end)
```

4. Solar Cell

4.1. Learning Objectives

The assignment of this unit is to investigate the behavior of a photovoltaic (PV) cell with different loads and illumination levels.
The students shall get to know the following features of photovoltaic energy harvesting:

- difference between the characteristics of a diode and of a solar cell
- *active action* and *passive reaction* of an active element
- *maximum obtainable power* (Leistungsanpassung) and *load adjustment* for non-linear sources

4.2. Theoretical Introduction

When not exposed to light photovoltaic cells behave nearly like common *solid state diodes* (Halbleiterdioden). The characteristics of these can be described by the formula (4.1). The signs of the voltage and the current according to the reference directions for voltage and current in the schematic of figure 4.1.

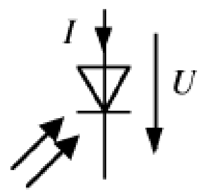


Figure 4.1.: Schematic of a Photovoltaic Cell
The double arrow symbolizing the light incidence.

$$I = I_S \left(e^{\frac{U}{U_T}} - 1 \right) \tag{4.1}$$

The equation (4.1) holds at constant temperature, I_S being the *reverse current* (Sperrstromstärke) and U_T a parameter given as a voltage depending mainly on the temperature.[1] Note that at zero voltage the current is zero too, which is a feature of *passive components*.

When exposed to light, the cell characteristics will shift in the direction of negative currents by an amount I_{phot} depending on the intensity of the light source (see equation (4.2) and

[1] The equation here is not rendered exactly in its usual form. This is of no importance here as the main aspects of the characteristic are covered so far.

figure 4.2). Part of the characteristics will then land in the 4th quadrant where energy can be gained from the cell when connected to a load: if the working point (Arbeitspunkt) of the cell lies in the 4th quadrant the sign of the energy flow (power) $P = UI$ is negative meaning that he cell delivers energy to the load.

$$I = -I_{phot} + I_S \left(e^{\frac{U}{U_T}} - 1 \right) \tag{4.2}$$

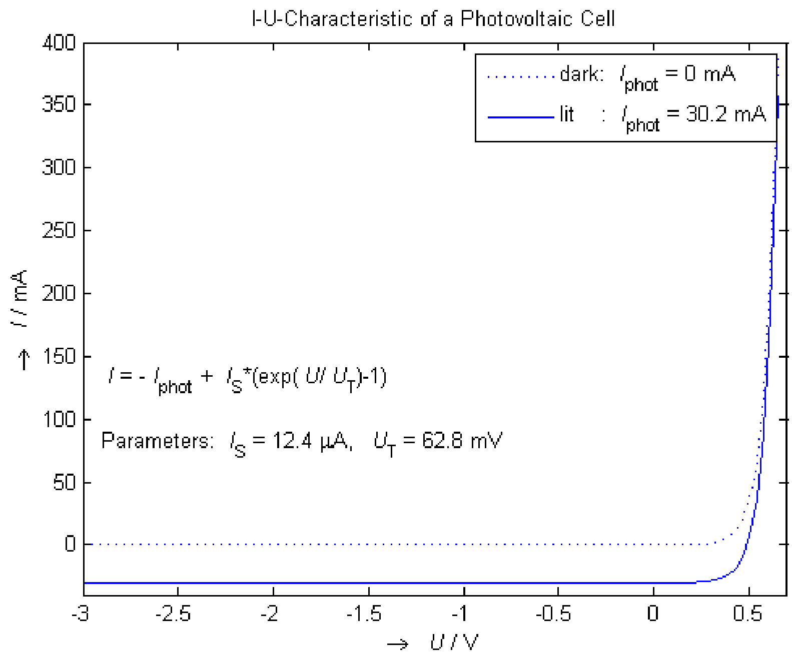


Figure 4.2.: Characteristics of a Photovoltaic Cell (dark and lit)

4.3. Practical Part

Before building the circuit according to the schematic of figure 4.3 the polarity of the photovoltaic cell should be determined by a simple voltage measurement: when exposed to light the cell should present a positive voltage in the direction of the reference arrow of the voltage U in figure 4.3. Once the circuit has been built, you should check for the unwanted influence of surrounding light on the cell and cover it with a cardboard box if necessary.

The characteristics of the PV cell should be measured in the 4th quadrant (positive voltages, negative currents) for two different levels of light intensity. This can be achieved through changing the electrical power of the light source.

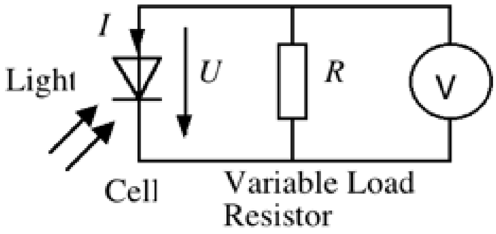


Figure 4.3.: Schematic for Measuring the Characteristics of a Photovoltaic Cell in the 4th Quadrant
For low values of the load resistor, the current cannot be measured with an ampere meter due to its internal resistance. In this case the current has to be determined with the voltage and the corresponding load resistance. Note that the current is negative in relation to the reference arrow in the schematic.
Due to the high resistance of the voltmeter the load current is practically the same as the one provided by the cell.

4.4. Evaluation

List of possible objectives of the experiment or related questions:

- The open circuit voltage of the PV cell is a measure for its temperature. Did this voltage change after exposure of the cell to light for some time? For this experience you should use a light bulb instead of LEDs in order to heat the cell.

- The short-circuit current of the PV cell is a measure for the intensity of the incoming light.[2] What are the values of the photovoltaic currents I_{phot} for the two levels of lightning? Did the choice of the light intensity make sense for your experiment?

- How does the maximal power change with the cell voltage for different intensities of the light? How much power does the PV cell deliver per cm^2 to the load under load adjustment conditions?

- How does the optimal load resistance change with the intensity of the light? How does it qualitatively have to be adapted to the lightning?

All the questions should be answered and commented based on the measurements. Furthermore *an estimation of measurement accuracy should be made for all the results.*

[2] In Switzerland the maximal obtainable radiation power is about 1 kW per square meter. The efficiency factor (Wirkungsgrad) of solar modules is typically between 15% and 20%.

5. DC Bridges

5.1. Learning Objectives

Measuring the resistance of different devices such as resistors, connection lines, contacts or even nonlinear components like diodes can be achieved with the help of DC measuring bridges (Gleichstrom-Messbrücken).
The students shall get to know the following features and application ranges of different DC bridges:

- calculating the equivalent circuit of bridge circuits with the measuring device as load
- operating mode and field of application of the Wheatstone bridge
- operating mode and field of application of the Thomson bridge
- the meaning of *sensitivity* (Empfindlichkeit) and *desadjustment* (Verstimmung) for a measuring bridge
- know the approximate low resistance range of electrical contacts and connection lines

5.2. Theoretical Introduction

There are two kinds of DC measuring bridges: the **Wheatstone** bridge[1] (Wheatstone-Brücke) for measuring values of resistances typically above $1\,\Omega$ and the **Thomsom** or **Kelvin** bridge[2] for values below $1\,\Omega$.

5.2.1. Wheatstone Bridge

The schematic of the Wheatstone DC bridge is shown in figure 5.1 on page 22. To balance the bridge the resistor R_N is trimmed until the voltage U_B or the current I_B across the measuring instrument (DMM) disappears. This condition leads to the following relation as both voltage dividers (Spannungsteiler) in the circuit show the same ratio:

$$I_B = 0 \quad \text{or} \quad U_B = 0 \qquad \rightarrow \qquad R_X = \frac{R_1}{R_2} R_N \tag{5.1}$$

However this condition cannot be achieved exactly in practice, as the resistor R_N cannot actually be adjusted perfectly to the required value. This detuning leads to a residual error in the calculation of R_X which is not accounted for in equation (5.1). This systematical error can be compensated mathematically with the sensitivity of the bridge, see equation (5.2).

1 *Charles Wheatstone* (1802 - 1875)
2 *William Thomson* alias *Lord Kelvin* (1824 - 1907)

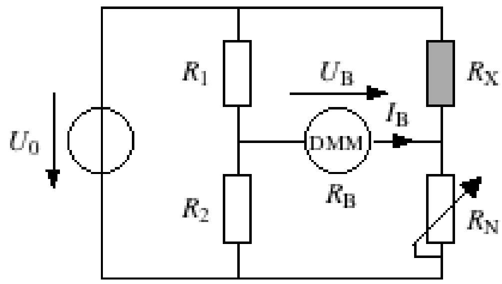


Figure 5.1.: Schematic of the Wheatstone DC Bridge
DMM stands for **d**igital **m**ulti**m**eter. This instrument can work either as a volt- or an ampere meter. The idea is to trim the resitance R_N until the voltage U_B or the current I_B is equal to zero. Under this condition the unknown resistance R_X can be calculated with equation (5.1). In order to give small measuring uncertainties, the values of the resistances R_1 and R_2 as well as R_N ans R_X should be approximately the same.

According to DIN 1319 the **sensitivity** (Empfindlichkeit) of a measuring device is defined by the ratio of the change in the reaction (displayed or recorded value of the DMM) to the change in its cause (variation in R_N). This sensitivity can be estimated by choosing two values for the resistor R_N close to the balance condition (left and right of it) and recording the corresponding values of U_B or I_B. This yields for I_B for example

$$S = \frac{\Delta I_B}{\Delta R_N} \approx \frac{I_{B2} - I_{B1}}{R_{N2} - R_{N1}} \tag{5.2}$$

Legend:

S	sensitivity, with unit that could be given here in μA/Ω (for U_B the unit of the sensitivity would be in μV/Ω)
ΔR_N	change in R_N round the balance condition (cause)
ΔI_B	corresponding change in I_B (reaction)

Apart from a practical approach the sensitivity could be determined by calculation. For that purpose it could be useful to work out the parameters U_E and R_E of the equivalent circuit of the bridge according to figure 5.2 using the equation pair (5.3).

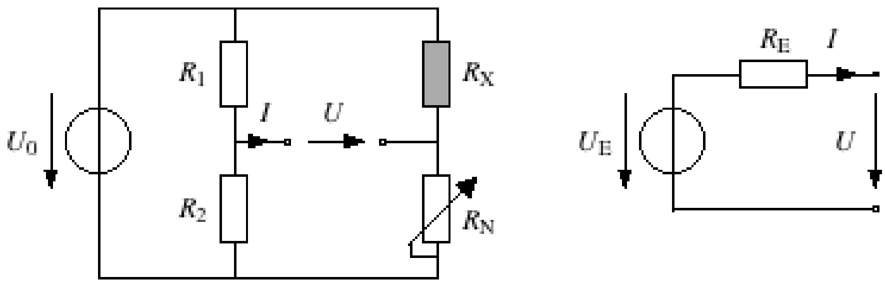


Figure 5.2.: Equivalent Circuit of the Wheatstone DC Bridge (without the measuring instrument)

$$\begin{aligned} R_E &= \frac{R_1\,R_2}{R_1+R_2} + \frac{R_X\,R_N}{R_X+R_N} \\ U_E &= \left(\frac{R_2}{R_1+R_2} - \frac{R_N}{R_X+R_N}\right) U_0 \end{aligned} \tag{5.3}$$

With the load resistor R_B accounting for the DMM the current I_B and the voltage U_B can be calculated with the following equations:

$$\begin{aligned} I_B &= \frac{U_E}{R_E+R_B} \\ U_B &= R_B\,I_B \end{aligned} \tag{5.4}$$

Notes

- The Wheatstone bridge is only suited for measuring the resistance of passive objects and certainly not for energized objects as for example the inner resistance of a voltage source!
- The bridge is not suited for measuring objects of low resistance as the resistance of all the connections and connecting lines of this object will be included in the result.
- Sometimes it is useful to know the conditions under which the resistance is measured, especially when the latter is dependent on the voltage or the current. In this case the influence of a possibly included measuring instrument should be analyzed carefully.

5.2.2. Thomson or Kelvin Bridge

The Thomson or Kelvin DC bridge is designed for measuring very low resistance values. Therefore to avoid the influence of the resistance of connecting lines and contacts the *force line* (stromführende Leitung) is separated from the *sense line* (Messleitung). The schematic of the Thomson or Kelvin DC bridge is shown in figure 5.3 on page 24. Eventually the double cursor of the two linear potentiometer R_P shall be trimmed (the value x indicating the relative position of the cursor pair: for $x=0$ the cursor pair is at its lowest, for $x=1$ at its top position.) until the current I_G across the measuring instrument (G: galvanometer) disappears. This condition leads to the same basic balance relation as in the case of the Wheatstone bridge. More precisely in the case of a double potentiometer:

$$I_G = 0 \qquad \rightarrow \qquad R_X = \frac{(1-x)\,R_P}{x\,R_P} R_N = \frac{1-x}{x} R_N \tag{5.5}$$

Legend:

R_X	unknown resistance of the object under test, shaded part without connections only
R_N	precision resistor with low resistance, e. g. $0.3\,\mathrm{m}\Omega$ or $30\,\mu\Omega$ (according to the data sheet in appendix A)
x	relative position of the cursor, defines the resistance ratio of the potentiometers: $R_1 = (1-x)\,R_P$ and $R_2 = x\,R_P$
I_0	measuring current from a current or voltage source, must usually be fairly high (e. g. several amperes) in order to induce a noticeable voltage over the object under test

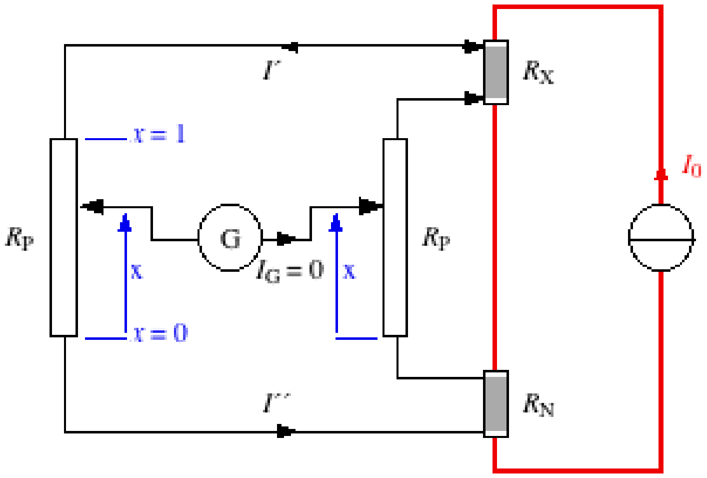


Figure 5.3.: Schematic of the Thomson or Kelvin DC Bridge
G stands for **g**alvanometer, that is a sensitive current measuring instrument. The idea is to simultaneously trim the two linear potentiometers R_P until the current I_G vanishes. This applies when the (weak) currents I' and I'' have the same strength. Under this condition the unknown resistance R_X (grey shaded part only) can be calculated with equation (5.5). The red loop shows the force line with high current.

5.3. Practical Part

5.3.1. Wheatstone Bridge

Make a mathematical model of the bridge using equations (5.3) and (5.4). This model will help you to design your own Wheatstone bridge with the resistors and the resistor decade in the lab. Use a DMM for measuring the current or the voltage of the bridge. Make your choice using the model to determine the best sensitivity.

Uncertainty in Measurement

From the equation of balance (5.1) and the rules of error propagation one can determine the uncertainty in measurement of R_x from the uncertainties of the other resistances:

$$\frac{u(R_X)}{R_X} = \sqrt{\left(\frac{u(R_1)}{R_1}\right)^2 + \left(\frac{u(R_2)}{R_2}\right)^2 + \left(\frac{u(R_N)}{R_N}\right)^2} \tag{5.6}$$

Legend:

$u(R_X)$	standard deviation of the uncertainty of the measured resistance the interval defined by the standard deviation $R_X \pm u(R_X)$ will cover 68% of the possibilities, the interval $R_X \pm 2u(R_X)$ up to 95%
$u(R_X)/R_X$	*relative* uncertainty of the measured resistance
$u(R_k)$	*absolute* uncertainty in R_k in Ω expressed as standard deviation
$= \Delta R_k/\sqrt{3}$	if the worst-case uncertainty intervall of R_k is $\pm\Delta R_k$

For a given value of $R_1 + R_2$ the relative uncertainty $u(R_X)/R_X$ is minimal for $R_1 = R_2$.

5.3.2. Thomson or Kelvin Bridge

As there is no double potentiometer available in the lab for building a Thomson bridge, you shall use the commercial device **Tettex Typ 2102** for the experiments. The corresponding data sheet can be found in appendix A.

Uncertainty in Measurement

According to the data sheet, the worst-case uncertainty in measurement (Garantiefehlergrenze) is 1% of the reading R_X. This corresponds to a standard deviation of

$$u(R_X) = \frac{0.01\,R_X}{\sqrt{3}}$$

assuming a uniform distribution density for the worst-case uncertainty.

5.4. Evaluation

List of possible objectives of the experiments or related questions:

Wheatstone Bridge

- How can the sensitivity of the bridge be increased in general? Are there any general rules for designing a Wheatstone bridge?
  - Determine whether measuring the voltage or the current with a DMM of a given model gives the higher sensitivity.
  - In which value range should the resistors R_1 and R_2 be chosen in order to give best results? Is there a relation to the value of the resistor under test?
- Show that the uncertainty in measurement deteriorates when the ratio between R_1 and R_2 or R_X and R_N gets fairly different from $1:1$.

Thomson or Kelvin Bridge

- How could the sensitivity of the bridge be increased?
- Does the quality of the contacts between the object under test and the connection to the potentiometers have an influence at all?
- What are the typical resistance values for wires and for different types of electrical connectors?

All the questions should be answered and commented based on the measurements. Furthermore *an estimation of measurement accuracy should be made for all the results.*

6. Mathematical Models of Nonlinear Characteristics

6.1. Learning Objectives

Elements like **diodes** or **varistors** (voltage dependent resistor, VDR, spannungsabhängiger Widerstand) show strongly nonlinear characteristics at constant temperature. These nonlinearities are conveniently used for different purposes as for rectifying alternating current in the case of diodes or for voltage limitation with diodes or varistors.
The students shall get to know the following issues about nonlinear characteristics:

- adequately describing measured nonlinear characteristics with simple mathematical exponential and power functions
- parameter fitting and validation of the chosen mathematical model

6.2. Theoretical Introduction

Sometimes it is useful to have a mathematical model for nonlinear characteristics of an electrical component in order to describe its behavior in a simulation model for example.

6.2.1. Diode

At constant temperature the voltage current characteristic of a common semiconductor diode (Halbleiterdiode) can be described by the following *exponential function*:

$$I = I_0 \cdot \left(\exp\left(\frac{U}{n\,U_T} \right) - 1 \right) \tag{6.1}$$

Legend:

I_0	*reverse (bias saturation) current* in A (Sperrstrom), typical value for Si: $< 1\,\mathrm{nA}$
U_T	*thermal voltage* in V (Thermospannung) typical value for Si at ambient temperature: about $26\,\mathrm{mV}$
n	*quality factor*, typically between 1 and 2 as U_T and n cannot be identified separately, the product $n\,U_T$ can be considered as a single parameter

Note: There is no sharp bend in this characteristic as one could expect from the usual simplified model of a diode used as a switch.

By taking the log (typically to the base 10), equation (6.1) can be simplified as follows:

$$\begin{aligned} \log I &= \log I_0 + \log\left(\exp\left(\frac{U}{n\,U_T} \right) - 1 \right) \\ \log I &\approx \log I_0 + \frac{\log e}{n\,U_T} U \qquad \text{for} \quad U > 5\,n\,U_T \end{aligned}$$

Plotted in a *semilogarithmic graph* this last equation will look linear. This particularity can help to confirm the exponential model for the function when measured points lay on a straight line. The parameters I_0 and $n\,U_T$ can directly be extracted from this linear representation.

6.2.2. Voltage Dependent Resistor or Varistor

Varistors can be used for over-voltage protection as they react almost instantly and their resistance increases rapidly with the applied voltage above a certain level. For this purpose the VDR must be connected in parallel to the object to protect.

The characteristic of a VDR can be approximated by the following *power function*:

$$\frac{U}{U_0} = \left(\frac{I}{I_0}\right)^{\alpha} \tag{6.2}$$

Legend:

I_0	reference current, can be chosen freely as long as I is expressed in the same unit
U_0	this parameter depends on the choice of I_0, unit in volts (the same as U)
α	exponent, typical value for a ZNR (zinc oxide resistor): $\alpha \approx 0.05$

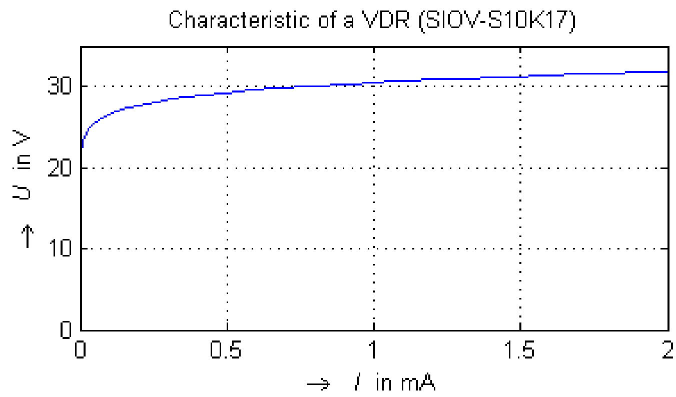


Figure 6.1.: Characteristic of a VDR

Taking the log of equation (6.2) yields:

$$\log\left(\frac{U}{U_0}\right) \quad = \quad \alpha \cdot \log\left(\frac{I}{I_0}\right)$$

Plotted in a *double logarithmic graph* this function will look linear. This particularity can help to confirm the power function model when measured points lay on a straight line. Choosing a convenient value for $I = I_0$ fixes the value of $U = U_0$. The parameter α can directly be extracted from the slope of this linear representation.

6.3. Practical Part

Measuring nonlinear characteristics requires some precaution in the choice of the measuring circuit as the systematical errors might dramatically change with the measuring conditions. For example the current and the resistance of a Diode change typically for more than 9 decades

for voltages between 0.1 V and 0.7 V, thus the influence of the resistance of the measuring instruments in the circuit has to be taken into account. As an other example see the R-I-characteristic of the VDR shown in figure 6.2.

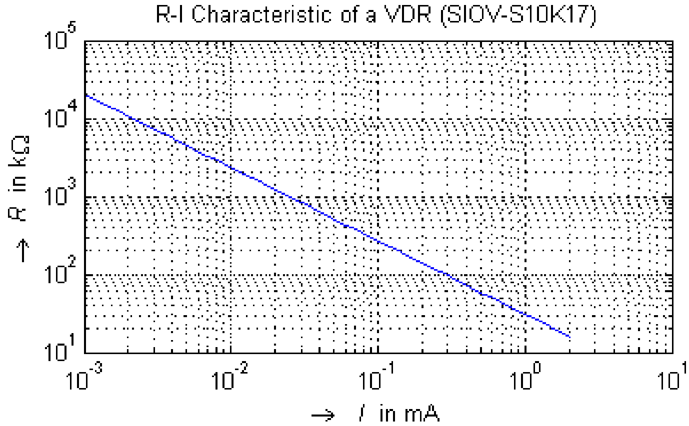


Figure 6.2.: R-I-Characteristic of a VDR
Note the range of the resistance of this object: from over 10 MΩ down to about 1 kΩ with increasing current.

Notes

- In order to find out the influence of the measuring current of a voltmeter it is best to disconnect the instrument and to observe possible changes in the current measured by the ampere meter (see figure 6.3 on page 29 for illustration). A noticeable change would suggest not to connect the voltmeter directly to the object under test, but over the ampere meter and the object connected in series.

- *A change in the scale of the ampere meter usually changes its internal resistance, also causing changes in the measured current.* That does not mean that one should not change to the most sensitive scale during measurement!

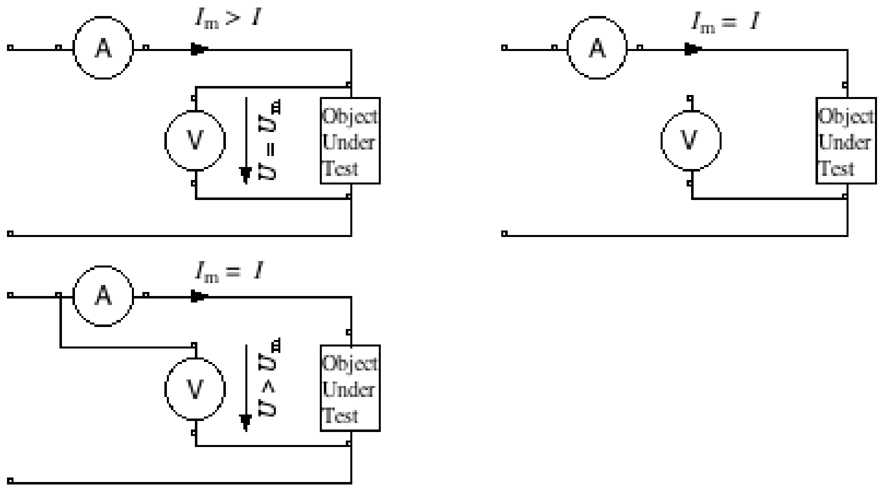


Figure 6.3.: Test for Systematical Error caused by Voltmeter and Countermeasure
Removing the voltmeter connection will possibly show a noticeable change in the current (see upper part of the figure). In this case it might be sensible to connect the voltmeter as shown in the lower schematic.

6.4. Evaluation

List of possible objectives of the experiments or related questions:

Diode

The following silicon diode type is available for experimentation:
1N4007, max. current 1 A

Assignment

- Measure simultaneously the voltage and the corresponding current of the diode in the first quadrant.
  Be careful to choose the correct measurement schematic in order not to distort the results with the resistances of the measuring instruments.
- Plot the characteristic with linear and semi-logarithmic scales. Identify the parameters of the mathematical model of the characteristic from these plots and find out how well the model fits the measurements in both the linear and logarithmic plots. Are the deviations due to uncertainty in measurement or to fundamental inaccuracies in the model?

Varistor

The following overvoltage protection varistor is available for experimentation:
EPCOS B72214 Typ S14K17, max. continuous power 100 mW

Assignment

Repeat the same measurements and evaluations as for the diode with the varistor. Use linear and double logarithmic scales for the representation of the characteristic.

All the questions should be answered and commented based on the measurements. Furthermore *an estimation of measurement accuracy should be made for all the results.*

7. Thermistors

7.1. Learning Objectives

Next to elements like diodes or varistors with nonlinear characteristics at constant temperature there exist elements whose U-I-characteristics depend strongly on the temperature. These dependency and the associated nonlinearities are conveniently used for different purposes as temperature measurement, current throttling (Stromdrosselung), overload protection or even passive power control.
The students shall get to know the following features and application ranges of thermistors, that is temperature dependent resistors:

- static and dynamical characteristics of thermistors and their applications
- thermistors as resistive temperature detectors (RTD)
  (see for example: http://en.wikipedia.org/wiki/Resistance_thermometer)

7.2. Theoretical Introduction

Thermistors are classified into two categories depending on their reaction to temperature: elements with **negative temperature coefficient**, **NTC** (Heissleiter) or with **positive temperature coefficient**, **PTC** (Kaltleiter). Both types are made up of semiconductor material (mainly metal oxides) an can therefore only be used up to temperatures of about 150 °C.

7.2.1. U-I Characteristic

As the temperature of thermistors plays an important role one has to differentiate between their static and dynamical characteristics:

Static Characteristic

The static U-I characteristic holds when the temperature of the thermistor has reached a *steady state* (Gleichgewichtszustand) which usually takes some time depending on the size of the element and the surrounding media, each operating point (Arbeitspunkt) on the characteristic corresponding to a specific temperature.

The temperature of the thermistor can be held constant, for example by immersing the element in a liquid (*external heating*), or be influenced by the current flowing in the thermistor itself (*self-heating*).[1] The external heating property allows the use of NTCs or PTCs as detectors for fluid levels. Examples of self-heating in air at ambient temperature are shown in the figures 7.1 and 7.2.

[1] Either a direct current (DC) or the RMS value (**r**oot **m**ean **s**quare, Effektivwert) of an alternating current (AC). The RMS value corresponding to the direct current (DC) that would produce the same real power in the element as the AC on the average.

Due to their steep increase in resistance over a given voltage level (see figure 7.3 on page 34) the PTCs can be used as passive and cheap overheat protection for electrical motors or loudspeakers for example. PTCs can also be used as control elements for constant power. This last property is apparent in figure 7.2: above a certain level the static characteristic is practically identical with the lines of constant power.

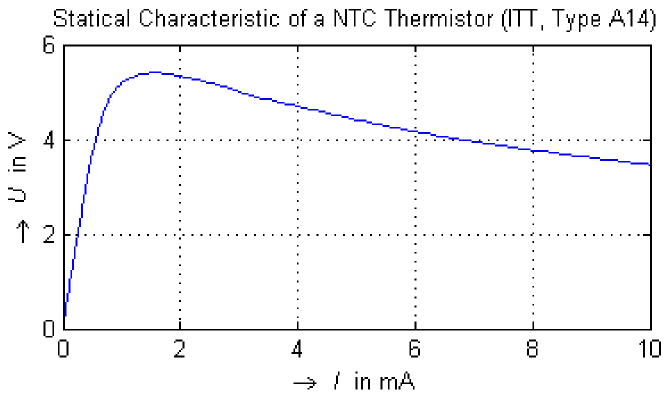


Figure 7.1.: Static (self-heating) U-I-Characteristic of a NTC Thermistor in Air at Ambient Temperature
The temperature of the NTC increases with the current flowing.
Note that connecting a NTC to an ideal voltage source can produce two operating points. The one at higher currents is unstable and will definitely overload the thermistor leading to its destruction. On the other hand an ideal current source or a linear source with high inner resistance leads to only one (controllable and stable) operating point.

Dynamical Characteristic

This characteristic holds when the rate of change in the applied voltage or current is fast compared to the change in temperature of the thermistor. In this case the temperature of the thermistor remains nearly constant. Therefore there exist a whole set of dynamical characteristics, every one corresponding to a given temperature. For NTCs the dynamical characteristics are linear, their slopes decreasing with temperature. For PTCs the characteristics are nonlinear, their slopes increasing with temperature.

7.2.2. R-T-Characteristic

With the operating points of the static U-I characteristic and their related temperatures one can derive a characteristic showing the temperature T in function of the corresponding resistance $R = \frac{U}{I}$ of the thermistor.

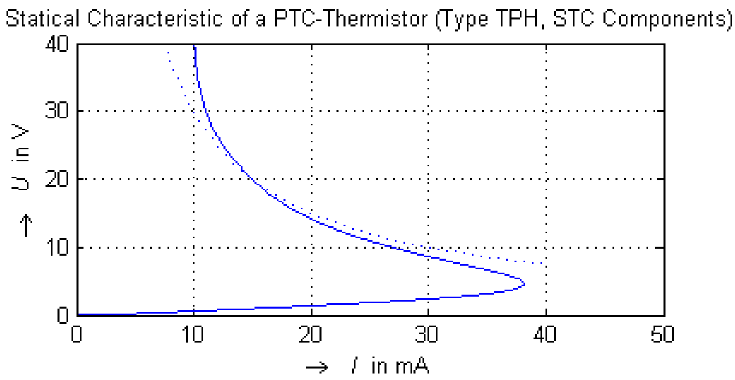


Figure 7.2.: Static (self-heating) U-I-Characteristic of a PTC Thermistor in Air at Ambient Temperature
The temperature of the PTC increases with the voltage applied.
Note that connecting a PTC to an ideal current source can produce two operating points. The one at higher voltages is unstable and will definitely overload the thermistor leading to its destruction. On the other hand an ideal voltage source leads to only one (controllable and stable) operating point.
Note as well that the characteristic is practically following the dotted line of constant power, here 0.3 W.

NTC

The sensitivity of the resistance of a NTC to temperature makes them ideal for temperature measurements. The static R-T-characteristic of an NTC can be described by the following equation:

$$R = R_0 \cdot \exp\left(\frac{B}{T} - \frac{B}{T_0}\right) = A \cdot \exp\left(\frac{B}{T}\right) \tag{7.1}$$

Legend:

T	absolute temperature in K (Kelvin)
	temperature in degrees Celsius: $\theta = T - 273\,°\mathrm{C}$
T_0	reference temperature in K
R_0	resistance in Ω at temperature T_0
B	material dependent constant in K, typical values between 2000 K and 5000 K
A	$= R_0 \cdot \exp\left(-\frac{B}{T_0}\right)$

A typical case is shown in figure B.1 on page 42. This highly nonlinear characteristic can more or less be linearized with a resistor parallel to the NTC. For a given temperature interval, lets assume from T_1 to T_2, the value of this resistance R_p should be

$$R_p = R_m \cdot \left(\frac{B - 2\,T_m}{B + 2\,T_m}\right) \tag{7.2}$$

Legend:

$T_m = \frac{T_1+T_2}{2}$	middle temperature of the interval in K
R_m	resistance of the NTC at $T = T_m$ according to equation (7.1) in Ω

PTC

The resistance of a PTC rises dramatically above a certain critical temperature (see figure 7.3).

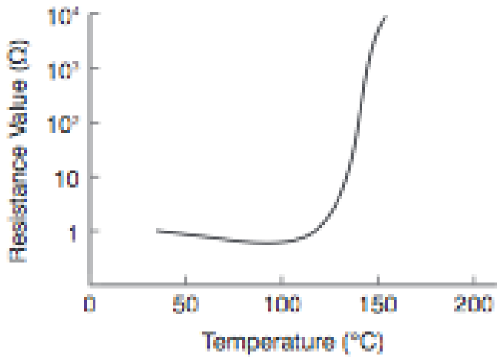


Figure 7.3.: Typical R-θ Characteristic of a PTC
(Source: http://www.murata.com/products/catalog/pdf/r16e.pdf)
Note that before the actual rise, the resistance might slightly sink with increasing temperature.

In the rising domain the static R-T-characteristic can be approximated by the following equation:

$$R \approx R_0 \cdot \exp\left(B\left(T - T_0\right)\right) \tag{7.3}$$

Legend:

T	absolute temperature in K (Kelvin)
T_0	reference temperature in K
R_0	resistance in Ω at temperature T_0
B	material dependent constant in K^{-1}

7.3. Practical Part

Measuring nonlinear characteristics requires some precaution in the choice of the measuring circuit as the systematical errors might dramatically change with the measuring conditions.

Temperature measurement

A NTC thermistor probe (Therp) is available for measuring the temperature of the objects under test (see data sheet B on page 40 and measuring schematic in figure 7.4). In order to obtain a good thermal contact between the object and the probe you should use some heat-conductive paste (Wärmeleitpaste). Don't use huge amounts of it and make sure to clean the probe and the object under test when finished.

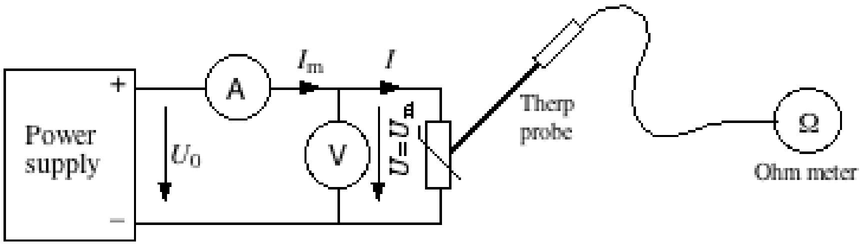


Figure 7.4.: Measurement of the Temperature of the Object under Test with the Therp Probe

7.4. Evaluation

List of possible objectives of the experiments or related questions:

7.4.1. NTC

- Temperature Detector

  EPCOS B57164, 4.7 kOhm, max. continuous power 450 mW, thermal time constant: $\tau = 20\,\mathrm{s}$

  - Measure the static characteristic of this NTC together with its temperature. *Be careful to control the measurement by limiting the current* (current source) *and not to use an ideal voltage source* (regulated DC power supply).
  - Plot the U-I-characteristic with linear scales with the temperature as parameter.
  - Plot $R = U/I$ versus θ characteristic with linear scales.
  - Plot R on a logarithmic scale versus $1/T$ on a linear scale. Identify the parameters of the mathematical model (7.1) from this plot and find out how well the model fits the measurements in both the linear and logarithmic plots. Are the deviations due to uncertainty in measurement or to fundamental inaccuracies in the model?

- Linearisation of the R-θ-Characteristic

  Therp probe

  - Choose a temperature interval for which the R-θ-characteristic of the probe shall be linearized and dettermine the resistance of the resistor according to equation (7.2) that will linearize the characteristic in that interval.
  - Connect the resistor in parallel to the probe and compare the result with the expectation using a second probe for reference.

- Current Throttling

  EPCOS Serie 235, according to DIN 400040 HGF, max. power at 25 °C: 1.8 W, max. resistance: 10 Ω, max. current 3 A

  - Build the schematic according to figure 7.5 and record the voltage $u(t)$ over the resistor (this is proportional to the current flowing) with a scope (Oszilloskop) after switching on the source. Explain the process. How long does it take to reach a steady state?

– Promptly reduce the voltage of the source to 5 V. What happens? How long does it take now to reach a steady state?

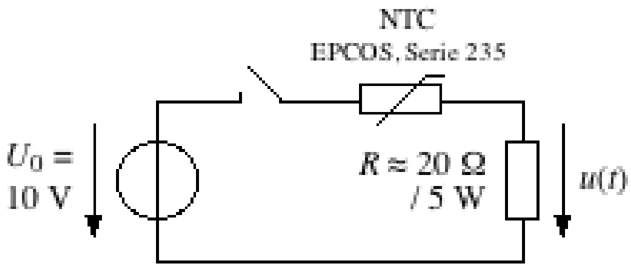


Figure 7.5.: Current Thottling with NTC
Limiting the rate of change of a current in a switching process is necessary with inductive loads to prevent voltage peaks. This can typically be done with a NTC.

7.4.2. PTC

The following PTCs is available for experimentation:
EPCOS B59008-C70-A40, $T_{nat} = 70\,°\mathrm{C}$

- Measure the static characteristic of a PTC together with its temperature.
  Be careful to control the measurement by limiting the voltage.
- Plot the U-I-characteristic with linear and double-logarithmic scales with the temperature as parameter.
- Which line of constant power follows practically the course of the characteristic?
- Plot $R = U/I$ versus θ characteristic with linear scales.

All the questions should be answered and commented based on the measurements. Furthermore *an estimation of measurement accuracy should be made for all the results.*

A. Tettex Typ 2102 – Data Sheet

This bridge has two modes of operation depending on the measurement range (see under A and B in the second column of the data sheet):

Measuring Range : 0.0009 to $1.1\,\Omega$

see schematic (A) on the second page of the data sheet
The sense line is attached to the connectors P. The force line enters in the back of the instrument and exits through the connectors J to the object under test. The resistance of interest is measured between the tips of the sense line (R_X in the schematic).

Note that the maximal current through the device shall never exceed $2\,\mathrm{A}$ in the present mode of operation.

Measuring Range : $< 0.0009\,\Omega$

see schematic (B) on the second page of the data sheet
For this measurements two external resistors are available:

- for the range 0.00009 to $0.0011\,\Omega$: $R_{N_{ext}} = 0.0003\,\Omega\,/\,\mathrm{max.}\;20\,\mathrm{A}$
- for the range 0.000009 to $0.00011\,\Omega$: $R_{N_{ext}} = 0.00003\,\Omega\,/\,\mathrm{max.}\;100\,\mathrm{A}$

In this mode of operation the force line is completely outside of the instrument. The resistance of interest is measured between the tips of the sense line (R_X in the schematic).

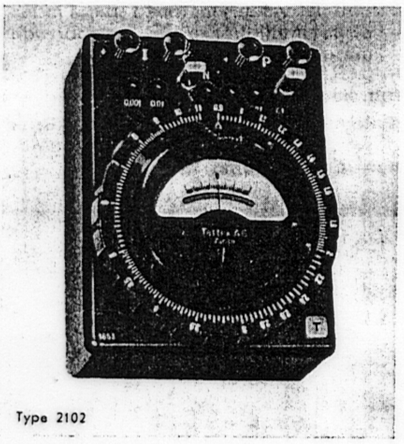
Note the polarity of the connection P of the sense line in the schematic and the formula for calculating R_X.

Note: The resistance should be calculated using the framed formula under B, that is

$$\begin{aligned} R_X = R_{N_{ext}} \frac{Reading}{3} &= 0.1\,\mathrm{m}\Omega \cdot Reading \quad \text{for} \quad R_{N_{ext}} = 0.0001\,\Omega \\ &= 10\,\mu\Omega \cdot Reading \quad \text{for} \quad R_{N_{ext}} = 0.00001\,\Omega \end{aligned}$$

For both modes of operation the worst case precision of the measurement is $\pm 1\%$ of the reading (Garantiefehlergrenze).

Schleifdraht-Widerstands-Meßbrücke Typ 2102



Type 2102

Meßbrücke nach Thomson Typ 2102

Ausführung

In handlichem, schwarzem, robustem Preßstoffgehäuse gemäß Abbildung. Schaltung nach Thomson mit Doppel-Schleifdraht.

Meßbereich

0.000 9 ... 1.1 Ohm,
unterteilt in 3 Einzel-Meßbereiche.
Durch zwei Stöpsel, die kleinste Uebergangs-Widerstände garantieren, können die drei Verhältnisse

× 0.001 - × 0.01 - × 0.1 (Ohm)
gewählt werden.

Eine Erweiterung des Meßbereiches für kleinere Widerstände ist durch extern anzuschließende Normalwiderstände (z.B. Typen 2117 und 2118) möglich.

Typ 2117: Für Meßbereiche × 0.000 1 Ω
Typ 2118: Für Meßbereiche × 0.000 01 Ω

Meßwert ablesbar an einer durchdrehbaren Ringskala von 280 mm Länge, beziffert von 0.9 bis 11.
Die Randwerte von 0 bis 0.9 und von 11 bis ∞ werden durch die Schleifdraht-Endwiderstände unterdrückt.

Meßgenauigkeit

± 1 % vom abgelesenen Wert
(± 1 mm auf der Ringskala)

Spannungsquelle

Als Speisung dient eine separate Stromquelle von 2 Volt (Akkumulator oder Gleichrichter-Gerät Typ 2113).
Die Meßbrücke besitzt Anschluß-Buchsen für max. 0.5 Amp. und max. 2 Amp. Meßstrom. (Eingebaute Strombegrenzungswiderstände.)

Nullinstrument

Präzisions-Drehspul-Galvanometer mit Messerzeiger und Spiegelskala.
Das Galvanometer wird bei der Messung mit dem Tasterschalter, der 2 Empfindlichkeitsstufen besitzt, eingeschaltet.

Bedienung

A) Messung von Widerständen von 0.000 9 bis 1.1 Ohm.

1. Unbekannten Widerstand R_X entsprechend Anschlußschema A an die Brücke anschließen. (Polarität beachten.) Widerstand der Potentialleitungen C max. 0.01 Ohm (z. B. 2.5 m von 4 mm²). Die Potentialabgriffe P an R_X müssen einwandfrei sein! Widerstand der Verbindungsleitung D max. ca. das Zehnfache von R_X.
2. Anschluß der Spannungsquelle an die Buchsen 2 oder 0.5 Amp. (je nach Belastbarkeit des zu messenden Widerstandes).
3. Mit den beiden Stöpseln entsprechenden Meßbereich (Verhältnis) wählen.
4. a) Tasterschalter T leicht drücken (reduzierte Empfindlichkeit) und Brücke durch Verdrehen des Schleifringes auf Galvanometer Null vorabgleichen.
   b) Tasterschalter T ganz durchdrücken und Brücke unter Drehen des Schleifringes genau abgleichen, bis der Zeiger des Galvanometers Null zeigt.

   Der Tasterschalter kann in der vollen Empfindlichkeitsstufe durch eine leichte Drehung nach rechts arretiert werden.
5. Meßwert am Index ablesen und mit dem gewählten Meßverhältnis multiplizieren.

B) Messung von Widerständen < 0.000 9 Ohm.

1. Unbekannten Widerstand R_X sowie Normalwiderstand R_N entsprechend Anschlußschema B an die Brücke anschließen. (Polarität beachten; siehe auch A 1.)
2. Anschluß der Spannungsquelle, sowie Vorwiderstand R_S mit eventuellem Kontrollinstrument A. (Strom entsprechend maximaler Belastung von R_N und R_X wählen.)
3. Einen Stöpsel in Stöpselsitz N stecken.

Weiteres Vorgehen gemäß A.

Berechnung von R_X:

$$\boxed{R_X = R_N \cdot \frac{\text{Ablesung}}{3}}$$

Externer Normalwiderstand Typ 2117:
0.000 3 Ohm (max. 20 Amp.) für Meßbereich × 0.000 1 Ohm.

Externer Normalwiderstand Typ 2118:
0.000 03 Ohm (max. 100 Amp.) für Meßbereich × 0.000 01 Ohm.

Unterhalt

Von Zeit zu Zeit Stöpsel und Stöpselsitze reinigen und mit reiner Vaseline einfetten.

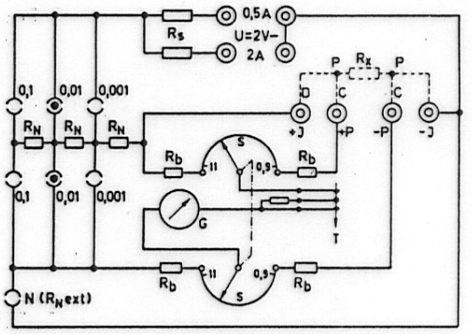
Dimensionen

120 × 160 × 80 mm Höhe

Nettogewicht

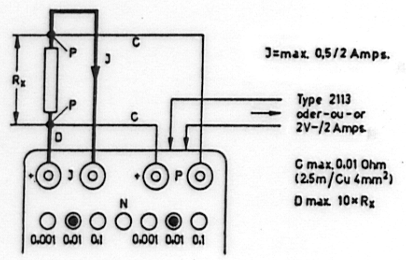
ca. 1.0 kg

Schema — Schéma de circuit — Circuit diagram
Type 2102

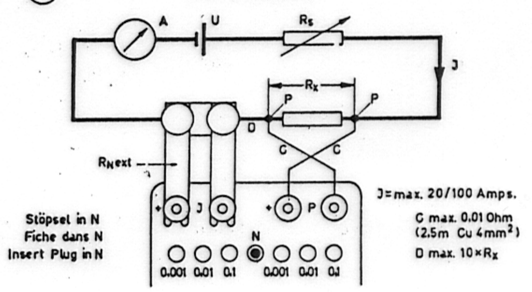


Anschlußschemata — Schémas de branchement — Junction diagrams
Type 2102

(A) R_X: 0.0009 ... 1.1 Ω



(B) R_X: < 0.0009 Ω



Legende

G = Galvanometer
S = Schleifdraht
R_X = Unbekannter Widerstand
R_V = Verhältnis-Widerstände
R_b = Schleifdraht-Endwiderstände
R_N = Normalwiderstände
R_N ext. = Externer Normalwiderstand
R_S = Strombegrenzungswiderstände
U = Spannungsquelle
T = Tasterschalter
C = Potentialleitungen
D = Verbindungsleitung
A = Kontrollinstrument
P = Potentialabgriffe

B. Thermistor Probe Therp – Data Sheet

The Therp probe is a NTC thermistor for determining temperatures between −50 °C and 250 °C through a resistance measurement. The conversion between resistance and temperature is given by a ruler with a double scale (see next page).

In the range from about 20 °C to 160 °C this conversion can be approximated by the following simple formula:[1]

$$\begin{aligned} T &= \frac{3300\,\mathrm{K}}{\ln\left(\frac{R}{0.126\,\Omega}\right)} \\ \theta &= T - 273\,\mathrm{K} \end{aligned} \tag{B.1}$$

This function is shown in the figure B.1.

Legend:

R	resistance of the NTC in Ω (Ohm)
T	temperature in K (Kelvin)
θ	temperature in °C (degrees centigrade)

The uncertainty of the Therp probe in function of the measured temperature can be approximated by the following equation:

$$\begin{aligned} \Delta\theta &= 1.15\cdot 10^{-4}\,{}^\circ\mathrm{C}^{-1}\,\theta^2 - 0.0075\,\theta + 1.6\,{}^\circ\mathrm{C} \qquad \text{(B.2)} \\ u(\theta) &= \frac{\Delta\theta}{\sqrt{3}} \end{aligned}$$

Legend:

$\pm\Delta\theta$	*worst case uncertainty* of measured temperature (Garantiefehlergrenze)
$u(\theta)$	uncertainty in θ expressed as *standard deviation*

The expression (B.2) only reflects the uncertainty of the sensor characteristic and does not include the uncertainty $u(R)$ in the measurement of its resistance. This can be done with the following relation:

$$\begin{aligned} u(\theta_{final}) &= \sqrt{\left(\frac{\mathrm{d}T}{\mathrm{d}R}\,u(R)\right)^2 + u(\theta)^2} \qquad \text{with} \qquad \frac{\mathrm{d}T}{\mathrm{d}R} = -\frac{1}{R}\frac{3300\,\mathrm{K}}{\left(\ln\frac{R}{0.126\,\Omega}\right)^2} \\ u(\theta_{final}) &\approx u(\theta) \qquad \text{if} \qquad \frac{u(R)}{R} < 1\% \end{aligned}$$

[1] This approximation is practically equivalent to equation (7.1) with the following parameters: $R_0 = 9.5\,\mathrm{k}\Omega$, $T_0 = 273\,\mathrm{K} + 20\,\mathrm{K} = 293\,\mathrm{K}$ and $B = 3300\,\mathrm{K}$

Therp

Any one of your testers
becomes "THERMOMETER"
when used with "THERP"!!

Description

"Therp" is used to determine temperature by measuring the change in the resistance value of the thermistor caused by temperature changes. Then, any type of testers, if within its resistance measuring range can be used for temperature measurement.

Features

1) High sensitive thermistor bead is fixed to the end of the probe.
2) Wide range of temperature, −50°C〜+250°C, can be determined.
3) Few time response, 5〜15 seconds in water.
4) Can be used in narrow place, small mass and any liquid.
5) Very simple to operate.

Specifications

1) Measuring range: −50°C〜+250°C, or −40°F〜+500°F
2) Time respose: Within 5 seconds (in water) Type 3S
   Within 15 seconds (in water) Type 3
3) Non polarity:
4) Accuracy: Within ± one scale of THERP CONVERSION SCALE.
   At 0°C ±1.6°C 100°C ±2.0°C
   25°C ±1.5°C 200°C ±4.7°C

Uses

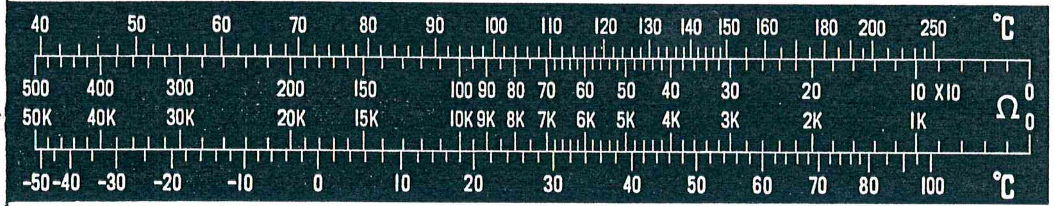
"Therp" can be used measuring temperature of such as follcwings:

General	Remoto Measuring
Electrical appliances	Seedbed
Soldering bath	etc.
Body	

Direction for Use

1) Use your tester as resistance measuring range.
2) Short circuit the (+) lead and (−) lead, adjust volume to indicate O ohm.
3) Pull out the tester probe and insert the "THERP" into (+) and (−).
4) Place the "THERP" aginst the object to be measured, then the meter will indicate the resistance of its temperature.
5) Convert the measured resistance into temperature by THERP CONVERT-ION SCALE.

for further information write to:



$$R = 0{,}126\,\Omega \cdot e^{\frac{3300\,K}{T}}$$

$$T = 273\,K + \vartheta$$

$$T = \frac{3300\,K}{\ln\left(R/0{,}126\,\Omega\right)}$$

$$\vartheta = T - 273\,K$$

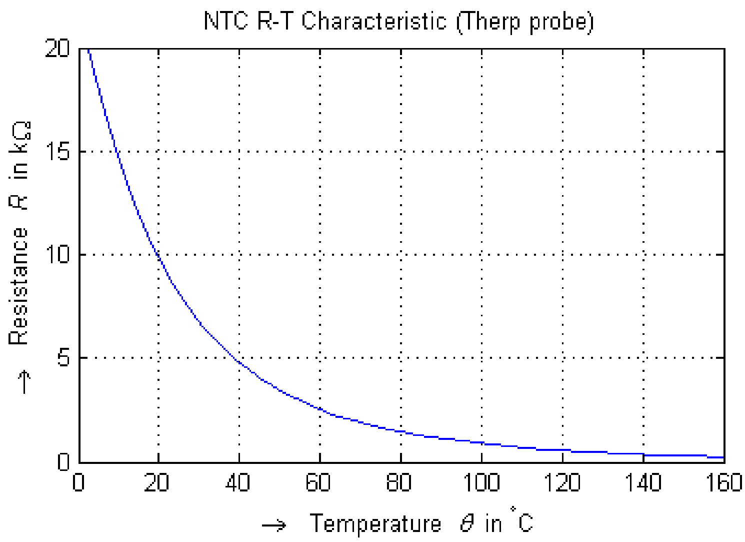


Figure B.1.: R-θ-Characteristic of the NTC Probe (Therp) according to equation (B.1)
Below 16 °C the mathematical model does not anymore fit the values according to the ruler in the data sheet.